Natural Convection Between Concentric Cylinders Using NEK 5000

Paul F. Fischer*and Yulia V. Peet†

May 1, 2009

1 Introduction

This note demonstrates the application of a spectral element code NEK 5000 to simulation of a natural convection. We consider a two-dimensional flow of air between the two concentric cylinders, with the inner cylinder slightly heated with respect to the outer one. Boussinesq approximation is used to formulate the equations of motion, valid in the situations where density differences are small enough to be neglected everywhere except in the gravitational forcing.

2 Governing Equations

The equations which govern a buoyancy-driven laminar motion in the Boussinesq approximation are

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 u + \alpha\, g\, (T - T_0) \tag{1}$$

$$\nabla u = 0 \tag{2}$$

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = k \nabla^2 T, \tag{3}$$

where u is the velocity vector, p is the pressure, ρ is the density, T is the temperature, ν is the kinematic viscosity, α is the volumetric thermal expansion coefficient, and g is the acceleration due to gravity. T_0 is the temperature of the outer cylinder, and ρ_0 is the corresponding fluid density, denoting the background quantities. Normalizing the above equations with L for the length scale (L is the width of the gap between the cylinders) and L/U for the time scale ($U \sim \sqrt{\alpha\, g\, L\, (T_1 - T_0)}$ is the characteristic velocity in the given problem),

*Mathematics and Computer Science Division, Argonne National Laboratory, fischer@anl.mcs.gov

†Mathematics and Computer Science Division, Argonne National Laboratory, peet@anl.mcs.gov

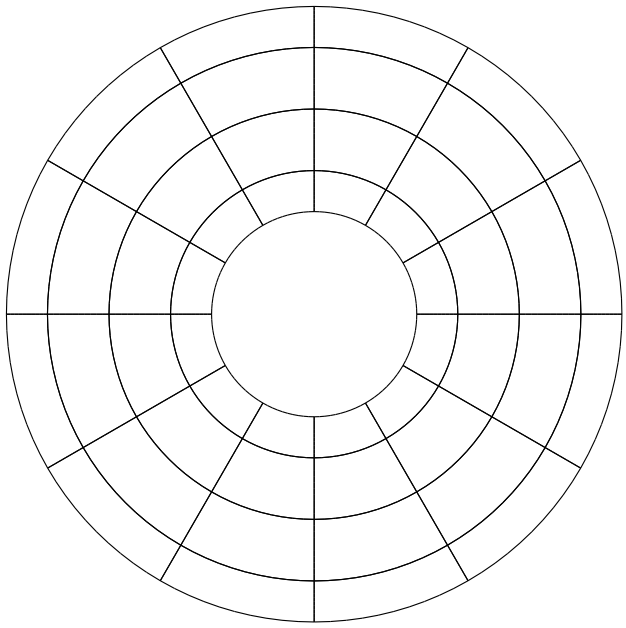

Figure 1: Spectral element mesh for the natural convection simulations.

and introducing non-dimensional temperature $\Theta = (T - T_0)/(T_1 - T_0)$, where T_1 is the temperature of the (heated) inner cylinder, the governing equations take the following non-dimensional form

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \frac{1}{\sqrt{Gr}} \nabla^2 u + \Theta \tag{4}$$

$$\nabla u = 0 \tag{5}$$

$$\frac{\partial \Theta}{\partial t} + u \cdot \nabla \Theta = \frac{1}{\sqrt{Gr}\, Pr} \Theta, \tag{6}$$

where two non-dimensional parameters were introduced

$$Gr = \frac{\alpha\, g\, (T_1 - T_0)\, L^3}{\nu^2}, \qquad Pr = \frac{k}{\nu}, \tag{7}$$

denoting Grashof number and Prandtl number, respectively.

3 Spectral Element Simulations

We have computed the steady-state flow between the concentric cylinders (inner cylinder is heated) with the two-dimensional computational mesh consisting of 4 elements in radial direction, and 12 elements in azimuthal direction. The order of elements is chosen to be 10 in the present simulations. Computational mesh

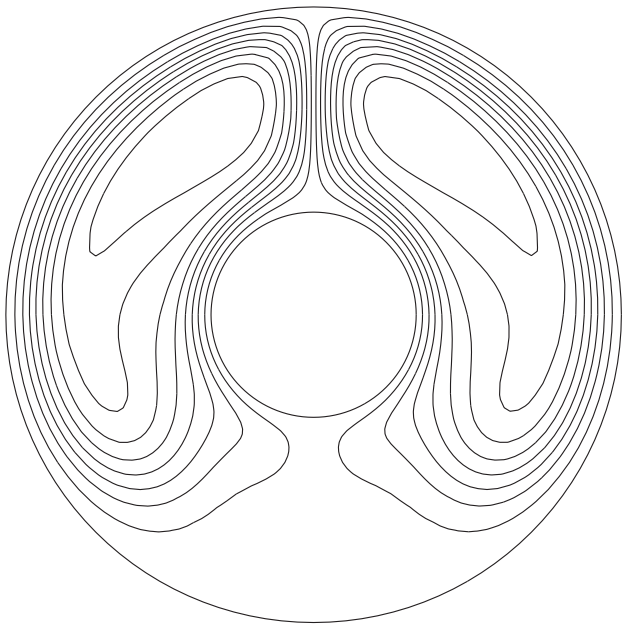

Figure 2: Streamlines in convection between concentric cylinders.

is shown in Fig. 1. The diameter of the outer cylinder is chosen to be three times the diameter of the inner cylinder, and the Grashof number is $Gr = 120,000$ to correspond to the parameters used in experiments of Grigull & Hauf [1] (visualization samples contained in the van Dyke's "Album of Fluid Motion" [2]), $Pr = 0.8$ is used for air. Simulations are run until the computational time $t\,L/U \sim 1000$, by which the steady-steady solution is obtained, indicated by the change in velocity magnitude $\sim 10^{-7}$ between the two successive time steps. The steady-state streamlines are visualized in Fig. 2, and isotherms – in Fig. 3 for the calculated case. The excellent agreement with the images of Grigull & Hauf [1] published in the book of Van Dyke [2] is evident.

References

[1] U. Grigull and W. Hauf. 1966. Proceedings of the 3rd International Heat Transfer Conference 2: 182-195.

[2] M. Van Dyke. *An Album of Fluid Motion.* The Parabolic Press, Stanford, California, 1982.

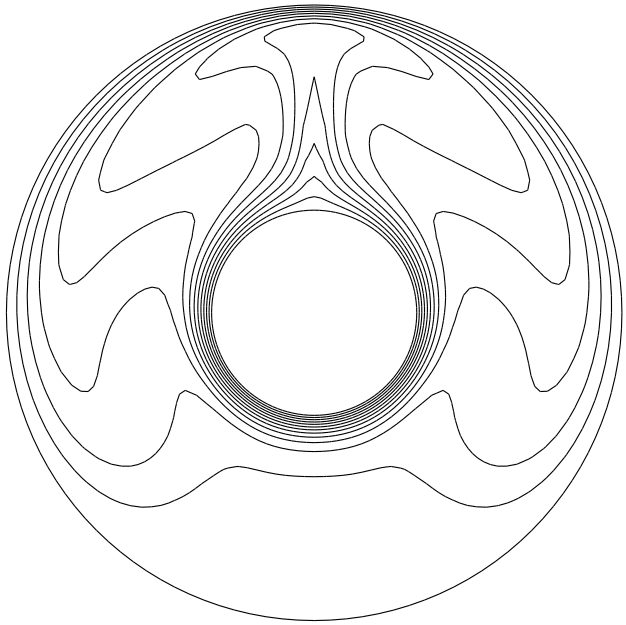

Figure 3: Isotherms in convection between concentric cylinders.